Self-consistent Fermi surface renormalization of two coupled Luttinger liquids

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Using functional renormalization group methods, we present a self-consistent calculation of the true Fermi momenta k_F^a (antibonding band) and k_F^b (bonding band) of two spinless interacting metallic chains coupled by small interchain hopping. In the regime where the system is a Luttinger liquid, we find that $\Delta = k_F^b - k_F^a$ is self-consistently determined by $\Delta = \Delta_1[1 + g_0^2 \ln(\Lambda_0/\Delta)^2]^{-1}$, where g_0 is the dimensionless interchain backscattering interaction, Δ_1 is the Hartree-Fock result for $k_F^b - k_F^a$, and $\Lambda_0 \gg \Delta$ is an ultraviolet cutoff. For $g_0^2 \ln(\Lambda_0/\Delta_1)^2 \gg 1$ even weak interachain backscattering leads to a strong reduction of the distance between the Fermi momenta.

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I. INTRODUCTION

What survives of the Luttinger liquid properties of a one-dimensional metal if two or more metallic chains are coupled by some weak interchain hopping $t_\perp$? Motivated by the proposal that the anomalous normal state properties of the high-temperature superconductors are a manifestation of a two-dimensional non-Fermi liquid state[1], many authors tried to find an answer to the above question, using non-perturbative methods such as bosonization or the renormalization group (RG)[2,3,4,5,6,7,8,9,10]. As a first step towards a solution of this rather difficult problem, it is instructive to study just two coupled spinless metallic chains[3,5,8,10].

In spite of many years of research, one fundamental aspect has only recently been addressed[10]: the *self-consistent* renormalization of the Fermi surface (FS). The band structure of two non-interacting metallic chains consists of a bonding and an antibonding band, leading in the generic case to a FS consisting of four points $\pm k_F^a$ and $\pm k_F^b$, as shown in Fig. 1. If the density n is held constant, then Luttinger's theorem implies that the sum $k_F^a + k_F^b = \pi n$ is not affected by interactions; however, the difference $\Delta = k_F^b - k_F^a$, which is proportional to $t_\perp$ in the absence of interactions, can be strongly renormalized. The FS renormalization in two coupled metallic chains has been studied previously using RG methods in Refs.[5,9,10]. However, the calculation of the FS in these works is either not self-consistent[5,9] in the sense discussed in the classic book by Nozières[11], or the self-consistency has not been properly implemented within the framework of the RG[10]. The self-consistent definition of the FS is follows naturally from the fact that the shape of the FS is a characteristic property of a zero temperature RG fixed point[12,13,14,15]. We emphasize that in our approach the concept of a flowing FS depending on the RG flow parameter[5,9] never appears: we construct the true FS as a RG fixed point, which by definition does not flow.

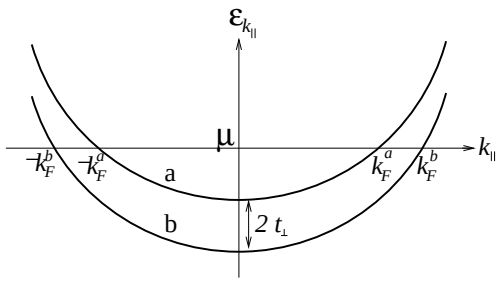

FIG. 1: Energy bands of two metallic chains with bare dispersions $\epsilon^a_{k_\parallel}$ (antibonding band) and $\epsilon^b_{k_\parallel}$ (bonding band). μ is the chemical potential. We consider the regime where the FS without interactions consists of four points $\pm k_F^a$ and $\pm k_F^b$. In the text k_F^a and k_F^b always refer to the true FS of the interacting system, defined via Eq. (3).

In this work we shall use our general method[12,13,14,15] of obtaining the FS as a RG fixed point to derive the self-consistency conditions for the true Fermi momenta k_F^a and k_F^b of two coupled spinless chains. In the parameter regime where the coupled chain system is a stable Luttinger liquid, these conditions can be cast into a simple transcendental equation for the distance $\Delta = k_F^b - k_F^a$ between the Fermi momenta, which for small $t_\perp$ and for weak interactions takes the form

$$\Delta = \Delta_1[1 + g_0^2 \ln(\Lambda_0/\Delta)^2]^{-1} \, . \tag{1}$$

Here Δ_1 is the value of $k_F^b - k_F^a$ within the Hartree-Fock approximation, g_0 is the bare value of a suitably defined (see below) dimensionless coupling describing interchain backscattering, and $\Lambda_0 \gg \Delta$ is an ultraviolet cutoff. We emphasize that Eq. (1) depends only logarithmically on Λ_0, so that the solution Δ is not very sensitive to the precise numerical value of Λ_0. Defining $x = \Delta_1/\Delta$ and $\gamma = g_0^2 \ln(\Lambda_0/\Delta_1)^2$, we may rewrite Eq. (1) in the form $x = 1 + \gamma + g_0^2 \ln x^2$, from which it is easy to see graphically that for $\gamma \gg 1$ even weak interchain backscattering leads to a strong reduction of the distance between the two Fermi momenta, although for finite γ they never merge. A numerical solution of Eq. (1) is shown in Fig. 2.

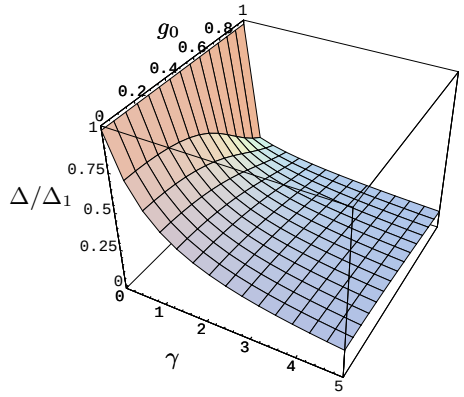

FIG. 2: Distance $\Delta = k_F^b - k_F^a$ between the Fermi momenta in the Luttinger liquid phase of two spinless chains coupled by weak interchain hopping $t_\perp$, see Eq. (1). Here Δ_1 is the value of $k_F^b - k_F^a$ within the Hartree-Fock approximation, g_0 is the bare dimensionless interchain backscattering interaction, and $\gamma = g_0^2 \ln(\Lambda_0/\Delta_1)^2$.

Before we discuss of derivation of Eq. (1), let us comment on its physical significance. First of all, Eq. (1) predicts a strong "attraction" between the two Fermi momenta. It is tempting to extrapolate this result to an infinite array of coupled chains: then the FS forms a continuum consisting of two corrugated sheets. Our two-chain result suggests that the curvature of the sheets should decrease due to interactions. This in turn enforces the nesting singularities and hence might stabilize a Luttinger liquid state even in higher dimensions. The fact that interchain back-scattering reduces the value of $k_F^b - k_F^a$ has already been noticed by Fabrizio[5]. However, he neither studied this phenomenon quantitatively, nor did he try to calculate the true FS self-consistently.

The renormalization of $k_F^b - k_F^a$ predicted by Eq. (1) behaves rather different from the estimate based on perturbation theory around the exact solution of the isolated chains with only forward scattering[7,16,17]. Such an estimate leads to the prediction that the renormalized interchain hopping is $t_\perp^* \propto t_\perp |t_\perp/v_F\Lambda_0|^{\frac{\eta}{1-\eta}}$, where η is the anomalous dimension of the Luttinger liquid and v_F the Fermi velocity for $t_\perp = 0$. The fact that without interactions $k_F^b - k_F^a \propto t_\perp$ suggests that in the interacting system $k_F^b - k_F^a \propto t_\perp^* \propto t_\perp[1 - \eta \ln|v_F\Lambda_0/t_\perp|]$ for small η. A comparison with Eq. (1) shows that this is not a valid procedure to estimate the location of the true FS.

II. EXACT RG EQUATION FOR THE FERMI SURFACE

In order to derive Eq. (1) we need to keep track of the momentum- and frequency dependence of the four-point vertex. To do this we use here the functional RG method[12,13,14,18,19,20,21,22,23]. We have derived Eq. (1) using both the field theoretical[13,14,21] and the Wilsonian formulation[12,22] of the functional RG. In order to make contact with previous work[5,9], let us outline here the derivation within the Wilsonian approach. We shall briefly comment on the corresponding calculation within the field theoretical RG in the concluding paragraph. Starting point in the Wilsonian approach is the exact hierarchy of RG flow equations for the irreducible self-energy $\Sigma_\Lambda(\sigma, K)$ and the one-particle irreducible $2n$-point vertices $\Gamma_\Lambda^{(2n)}(\{\sigma_i, K_i\})$ describing the change of these quantities as we vary the infrared cutoff Λ. Here $K = (k_\parallel, i\omega)$ denotes the momentum $k_\parallel$ along the chain and the frequency $i\omega$. The pseudospin $\sigma = a, b$ labels the two bands. We use the Matsubara formalism at zero temperature and the same sign conventions for the vertices as in Ref. [23]. For convenience we introduce the cutoff Λ in momentum space, so that for a given Λ all states with $|k_\parallel \pm k_F^\sigma| > \Lambda$ are integrated out.

To take the scaling properties close to the Luttinger liquid fixed point into account, we introduce rescaled momenta q and frequencies $i\epsilon$ by setting $k_\parallel = \alpha k_F^\sigma + \Lambda q$ and $i\omega = v_F \Lambda i\epsilon$, where $\alpha = \mathrm{sgn} k_\parallel = \pm 1$ labels the right/left half-axis. We define the rescaled two-point vertex by

$$\tilde{\Gamma}_l^{(2)}(\sigma, \alpha, Q) = \frac{Z_l^\sigma}{v_F \Lambda}[\Sigma_\Lambda(\sigma, K) - \Sigma(\sigma, \alpha k_F^\sigma, i0)] \; , \quad (2)$$

which is a function of the rescaled energy-momentum $Q = (q, i\epsilon)$ and the logarithmic flow parameter $l = \ln(\Lambda_0/\Lambda)$. Here Z_l^σ is the wave-function renormalization factor, and the counter-term $\Sigma(\sigma, \alpha k_F^\sigma, i0)$ is the exact self-energy at the exact k_F^σ. Once we know the counter-term in Eq. (2), we can calculate the true FS for a given chemical potential μ from the defining equation[11]

$$\epsilon_{k_F^\sigma}^\sigma + \Sigma(\sigma, \alpha k_F^\sigma, i0) = \mu \; , \quad (3)$$

where $\epsilon_{k_\parallel}^\sigma$ is the energy dispersion of band $\sigma = a, b$. In higher dimensions the corresponding counter-term $\Sigma(\mathbf{k}_F, i0)$ and the momenta $\mathbf{k}_F$ on the FS can be calculated within self-consistent perturbation theory in the weak coupling regime[24]. On the other hand, in one dimension it is necessary to invoke the RG to properly treat the infrared divergencies. A formally exact equation for the counter-term can be obtained from the requirement that for $l \to \infty$ the relevant part

$$r_l^\sigma \equiv \tilde{\Gamma}_l^{(2)}(\sigma, \alpha, 0) \quad (4)$$

of the two-point vertex (2) flows into a RG fixed point[15]. Defining the rescaled four-point vertex via

$$\tilde{\Gamma}_l^{(4)}(\sigma_1'\alpha_1'Q_1', \sigma_2'\alpha_2'Q_2'; \sigma_2\alpha_2 Q_2, \sigma_1\alpha_1 Q_1) \; = \; (2\pi v_F)^{-1} \times \Big(\prod_{i=1,2} Z_l^{\sigma_i'} Z_l^{\sigma_i}\Big)^{1/2} \Gamma_\Lambda^{(4)}(\sigma_1' K_1', \sigma_2' K_2'; \sigma_2 K_2, \sigma_1 K_1) \; , \quad (5)$$

the exact RG flow equation for $\tilde{\Gamma}_l^{(2)}(\sigma, \alpha, Q)$ reads

$$\partial_l \tilde{\Gamma}_l^{(2)} = (1 - \eta_l^\sigma - Q \cdot \partial_Q)\tilde{\Gamma}_l^{(2)} + \dot{\Gamma}_l^{(2)}(\sigma\alpha Q) \; , \quad (6)$$

where $\eta_l^\sigma = -\partial_l \ln Z_l^\sigma$ is the flowing anomalous dimension, $Q\cdot\partial_Q = q\partial_q + \epsilon\partial_\epsilon$, and the inhomogeneity is

$$\dot{\Gamma}_l^{(2)}(\sigma\alpha Q) = -\sum_{\sigma'\alpha'}\int \frac{dq'd\epsilon'}{(2\pi)^2}\dot{G}_l(\sigma'\alpha'Q') \times\tilde{\Gamma}_l^{(4)}(\sigma\alpha Q,\sigma'\alpha'Q';\sigma'\alpha'Q',\sigma\alpha Q)\ . \quad (7)$$

Here for sharp cutoff and linear dispersion

$$\dot{G}_l(\sigma\alpha Q) = -\delta(1-|q|)/[Z_l^\sigma(i\epsilon-\alpha v_0^\sigma q)-\tilde{\Gamma}_l^{(2)}(\sigma\alpha Q)]\ , \quad (8)$$

with the dimensionless Fermi velocities $v_0^\sigma = v_F^\sigma/v_F$, where v_F^σ is the bare Fermi velocity associated with band σ. Because the coupling r_l^σ is relevant with scaling dimension +1, its initial value r_0^σ must to be fine tuned in order to approach a finite limit for $l\to\infty$. This leads to the condition[15]

$$r_0^\sigma = -\int_0^\infty dl e^{-l+\int_0^l dt\eta_t^\sigma}\dot{\Gamma}_l^{(2)}(\sigma,\alpha,0)\ , \quad (9)$$

which relates the inital value r_0^σ to the RG flow on the entire RG trajectory. Assuming that Λ_0 is sufficiently large so that Σ_{Λ_0} can be neglected and $Z_0^\sigma\approx 1$, we obtain from Eq. (2) for the counter-term

$$\Sigma(\sigma,\alpha k_F^\sigma,i0) = -v_F\Lambda_0 r_0^\sigma\ . \quad (10)$$

At constant density $n=(k_F^a+k_F^b)/\pi$ we then find

$$k_F^b - k_F^a \equiv \Delta = \Delta_0 + 2\Lambda_0(v_0^b+v_0^a)^{-1}(r_0^b-r_0^a)\ , \quad (11)$$

where Δ_0 is the value of $k_F^b-k_F^a$ at the same density but without interactions.

III. WEAK COUPLING TRUNCATION FOR THE EFFECTIVE INTERACTION AND THE SELF-ENERGY

The formally exact Eqs. (6–11) are now our starting point to calculate Δ self-consistently. Note that the concept of a "flowing FS" never appears in our approach; by definition, k_F^a and k_F^b are the true Fermi momenta of the interacting system, to be determined self-consistently from Eqs. (9) and (11). To make further progress, we need an approximate expression for the effective interaction $\tilde{\Gamma}_l^{(4)}$ in Eq. (7). To simplify the analysis, we neglect vertices describing intrachain umklapp scattering as well as chiral vertices, involving particles moving in the same direction. The latter are expected to give finite renormalizations of the Fermi velocities, which we ignore here. Totally we should then keep track of the RG flow of the following five vertex functions,

$$\tilde{f}_l^{bb}(Q_1',Q_2';Q_2,Q_1) = \tilde{\Gamma}_l^{(4)}(bQ_1',bQ_2';bQ_2,bQ_1), \quad (12a)$$
$$\tilde{f}_l^{aa}(Q_1',Q_2';Q_2,Q_1) = \tilde{\Gamma}_l^{(4)}(aQ_1',aQ_2';aQ_2,aQ_1), \quad (12b)$$
$$\tilde{f}_l^{ab}(Q_1',Q_2';Q_2,Q_1) = \tilde{\Gamma}_l^{(4)}(aQ_1',bQ_2';bQ_2,aQ_1), \quad (12c)$$
$$\tilde{u}_l(Q_1',Q_2';Q_2,Q_1) = \tilde{\Gamma}_l^{(4)}(bQ_1',bQ_2';aQ_2,aQ_1), \quad (12d)$$
$$\tilde{g}_l(Q_1',Q_2';Q_2,Q_1) = \tilde{\Gamma}_l^{(4)}(bQ_1',aQ_2';bQ_2,aQ_1), \quad (12e)$$

where on the right-hand side the direction labels are $(\alpha_1',\alpha_2';\alpha_2,\alpha_1) = (+,-;-,+)$. Here $\tilde{f}_l^{\sigma\sigma'}$ describes forward scattering of two fermions associated with bands σ and σ', $\tilde{u}_l$ describes interband umklapp scattering (also called pair tunneling), and $\tilde{g}_l$ corresponds to interband backscattering. The functional RG equations for these five functions follow from the general flow equations given in Ref.[12] in a straightforward way. If we set $Q_i = Q_i' = 0$ and denote the corresponding coupling *constants* by $f_l^{\sigma\sigma'}$, u_l and g_l (without a tilde), we obtain the one-loop RG flow equations

$$\partial_l u_l = -u_l f_l\ ,\ \partial_l g_l = g_l f_l\ ,\ \partial_l f_l = -Au_l^2 + Bg_l^2\ , \quad (13)$$

where

$$f_l = f_l^{aa}/v_0^a + f_l^{bb}/v_0^b - 2f_l^{ab}/\bar{v}_0\ , \quad (14)$$

with $A = 2/(v_0^a v_0^b) + 2/\bar{v}_0^2$ and the average dimensionless velocity $\bar{v}_0 = (v_0^a+v_0^b)/2$. The function

$$B_l = 2C_l/(v_0^a v_0^b) + 2\bar{C}_l/\bar{v}_0^2 \quad (15)$$

depends on the rescaled Fermi point difference $\tilde{\Delta}_l = \Delta e^l/\Lambda_0$ via

$$C_l = \Theta(e^l-1-2|\tilde{\Delta}_l|)/(1+|\tilde{\Delta}_l|)\ , \quad (16)$$
$$\bar{C}_l = \Theta(e^l-1-2|\tilde{\Delta}_l|)\frac{1}{2}\sum_\sigma \frac{\bar{v}_0}{\bar{v}_0+v_0^\sigma|\tilde{\Delta}_l|}\ . \quad (17)$$

The RG equations (13) are equivalent to those derived by Fabrizio[5] at one-loop order. Note that the RG flow of u_l and g_l couples to the forward scattering interactions only via the combination f_l defined above. The flow of $f_l^{\sigma\sigma'}$ is

$$v_0^a\partial_l f_l^{bb} = -u_l^2 + C_l g_l^2 = v_0^b\partial_l f_l^{aa}\ , \quad (18)$$
$$\bar{v}_0\partial_l f_l^{ab} = u_l^2 - \bar{C}_l g_l^2\ . \quad (19)$$

As discussed by Fabrizio[5], the above RG equations predict a finite regime of initial values $f_0^{\sigma\sigma'}$, u_0, and g_0 where the system is a stable Luttinger liquid, characterized by finite forward scattering couplings $f_l^{\sigma\sigma'}$ and vanishing u_l and g_l for $l\to\infty$.

We now calculate the true FS in the Luttinger liquid regime using our general Eqs. (6–11). Assuming that $f_l^{\sigma\sigma'}$, u_l, and g_l are small compared with unity, we may adopt the same strategy as in Refs.[22,25]: we expand the right-hand side of the flow equations for the momentum- and frequency dependent vertices defined in Eqs. (12a–12e) to second order in $f_l^{\sigma\sigma'}$, u_l, and g_l. The flow of the above vertex functions is then given by the following set of equations,

$$D_l^{bb}\tilde{f}_l^{bb}(Q_1',Q_2';Q_2,Q_1) = (f_l^{bb})^2[\dot{\chi}_l^{bb}(Q_1-Q_2') - \dot{\chi}_l^{bb}(Q_1+Q_2)] - u_l^2\dot{\chi}_l^{aa}(Q_1+Q_2) + g_l^2\dot{\chi}_l^{aa}(Q_1-Q_2'+2\tilde{\Delta}_l)\ , \quad (20a)$$

$$D_l^{aa}\tilde{f}_l^{aa}(Q_1',Q_2';Q_2,Q_1) \;=\; (f_l^{aa})^2[\dot{\chi}_l^{aa}(Q_1-Q_2')-\dot{\chi}_l^{aa}(Q_1+Q_2)] -u_l^2\dot{\chi}_l^{bb}(Q_1+Q_2)+g_l^2\dot{\chi}_l^{bb}(Q_1-Q_2'-2\tilde{\Delta}_l)\;, \quad (20b)$$

$$D_l^{ab}\tilde{f}_l^{ab}(Q_1',Q_2';Q_2,Q_1) \;=\; (f_l^{ab})^2[\dot{\chi}_l^{ab}(Q_1-Q_2')-\dot{\chi}_l^{ab}(Q_1+Q_2)] +u_l^2\dot{\chi}_l^{ba}(Q_1-Q_2')-g_l^2\dot{\chi}_l^{ba}(Q_1+Q_2-2\tilde{\Delta}_l)\;, \quad (20c)$$

$$D_l^{ab}\tilde{u}_l(Q_1',Q_2';Q_2,Q_1) \;=\; f_l^{ab}u_l[\dot{\chi}_l^{ab}(Q_1-Q_2')+\dot{\chi}_l^{ba}(Q_1-Q_2')] -f_l^{bb}u_l\dot{\chi}_l^{bb}(Q_1+Q_2)-f_l^{aa}u_l\dot{\chi}_l^{aa}(Q_1+Q_2)\;, \quad (20d)$$

$$D_l^{ab}\tilde{g}_l(Q_1',Q_2';Q_2,Q_1) \;=\; -f_l^{ab}g_l[\dot{\chi}_l^{ab}(Q_1+Q_2)+\dot{\chi}_l^{ba}(Q_1+Q_2)] +f_l^{bb}g_l\dot{\chi}_l^{bb}(Q_1-Q_2')+f_l^{aa}g_l\dot{\chi}_l^{aa}(Q_1-Q_2')\;. \quad (20e)$$

Here $Q+\tilde{\Delta}_l$ is a short notation for $(q+\tilde{\Delta}_l,i\epsilon)$, the generalized susceptibilities $\dot{\chi}_l^{\alpha\beta}(Q)$ are given by

$$\dot{\chi}_l^{\alpha\beta}(Q) \;=\; \frac{\Theta(e^l-1-|q|)}{2\bar{v}_0+v_0^\alpha|q|-i\epsilon\mathrm{sgn}q} \;+\; \frac{\Theta(e^l-1-|q|)}{2\bar{v}_0+v_0^\beta|q|+i\epsilon\mathrm{sgn}q}\;, \quad (21)$$

where $\alpha,\beta\in\{a,b\}$, and

$$D_l^{\alpha\beta}=\partial_l+\eta_l^\alpha+\eta_l^\beta+\sum_{i=1}^{2}(Q_i\cdot\partial_{Q_i}+Q_i'\cdot\partial_{Q_i'}) \quad (22)$$

are comoving derivatives. The linear partial differential equations (20a–20e) can easily be solved exactly[22], so that we can obtain an explicit expression for the flowing momentum- and frequency dependent effective interaction $\tilde{\Gamma}_l^{(4)}(\sigma\alpha Q,\sigma'\alpha'Q';\sigma'\alpha'Q',\sigma\alpha Q)$. Substituting this expression into Eq. (7) and performing the Q'-integration we find

$$\begin{aligned}\dot{\Gamma}_l^{(2)}(a,+,Q) \;=\;& f_l^{aa}+f_l^{ab}-\partial_l[f_l^{aa}+f_l^{ab}] \\ &-2(f_l^{aa})^2I_{a,l}^{aa}(Q)-2(f_l^{ab})^2I_{b,l}^{ab}(Q) \\ &-u_l^2[I_{a,l}^{bb}(Q)+I_{b,l}^{ba}(Q)] \\ &-g_l^2[I_{a,l}^{bb}(q-2\tilde{\Delta}_l,i\epsilon)+I_{b,l}^{ba}(q-2\tilde{\Delta}_l,i\epsilon)]\;, \end{aligned} \quad (23)$$

where for $\alpha,\beta,\gamma\in\{a,b\}$ the integrals $I_{\gamma,l}^{\alpha\beta}(Q)$ are

$$\begin{aligned}I_{\gamma,l}^{\alpha\beta}(Q) \;=\;& \frac{1}{v_0^\alpha+v_0^\beta}\sum_{n=\pm1}n\Theta(e^l-1-|q+n|) \\ \times&\left\{\Theta(ns_{q+n})\ln\left[\frac{e^l(v_0^\alpha+v_0^\beta)+v_0^\gamma-v_0^\beta|q+n|-in\epsilon}{v_0^\alpha+v_0^\beta+v_0^\gamma+v_0^\alpha|q+n|-in\epsilon}\right]\right. \\ &\left.+\Theta(-ns_{q+n})\ln\left[\frac{e^l(v_0^\alpha+v_0^\beta)+v_0^\gamma-v_0^\alpha|q+n|-in\epsilon}{v_0^\alpha+v_0^\beta+v_0^\gamma+v_0^\beta|q+n|-in\epsilon}\right]\right\}, \end{aligned} \quad (24)$$

with $s_{q+n}=\mathrm{sgn}(q+n)$. The function $\dot{\Gamma}_l^{(2)}(b,+,Q)$ can be obtained from Eq. (23) by replacing $a\leftrightarrow b$ and $\tilde{\Delta}_l\to-\tilde{\Delta}_l$ on the right-hand side. Once we know the function $\dot{\Gamma}_l^{(2)}(\sigma,+,Q)$, we may calculate the flowing anomalous dimension from[22]

$$\eta_l^\sigma=-\partial\dot{\Gamma}_l^{(2)}(\sigma,+,Q)/\partial(i\epsilon)|_{Q=0}\;. \quad (25)$$

We can reproduce the result by Louis *et al.*[9] if we simply set $\dot{\Gamma}_l^{(2)}(a,+,Q)\approx f_l^{aa}+f_l^{ab}$ instead of Eq. (23), which amounts to replacing the effective interaction $\tilde{\Gamma}_l^{(4)}(\sigma\alpha Q,\sigma'\alpha'Q';\sigma'\alpha'Q',\sigma\alpha Q)$ in Eq. (7) by its value at $Q=Q'=0$. This approximation does not consistently take into account all contributions to r_0^σ which are quadratic in the bare couplings, because such second order terms are also generated by the Q'-dependence of the four-point vertex in Eq. (7).

To calculate the FS from Eq. (11), we still need to perform the trajectory integral (9). Since we are interested in the leading behavior for small $t_\perp$, we may at this point ignore the small difference between the bare Fermi velocities, setting $v_0^a=v_0^b=\bar{v}_0=1$. To second order in the couplings, we may also set $\eta_t^\sigma=0$ in Eq. (9). With these approximations the integral obtained by substituting Eq. (23) into Eq. (9) can be performed analytically to second order in the bare couplings. For $\tilde{\Delta}\equiv\Delta/\Lambda_0\ll1$ the result is $r_0^a=-f_0^{aa}-f_0^{ab}-g_0^2\tilde{\Delta}\ln|\tilde{\Delta}|$ and $r_0^b=-f_0^{bb}-f_0^{ba}+g_0^2\tilde{\Delta}\ln|\tilde{\Delta}|$ to leading logarithmic order. Substituting this into Eq. (11) we arrive at Eq. (1), with $\Delta_1=\Delta_0-\Lambda_0(f_0^{bb}-f_0^{aa})$, where we have used $f_0^{ab}=f_0^{ba}$.

Finally, let us point out that we have also derived Eq. (1) within the field-theoretical RG, where we calculate the renormalized vertex functions directly from perturbation theory. Setting again $v_0^a=v_0^b=1$ we obtain for the difference between the self-energies $\Sigma_\sigma=\Sigma(\sigma,k_F^\sigma,i0)$ at the true FS, up to two-loop order,

$$\Sigma_b-\Sigma_a=-2t_\perp+v_F\Lambda_0[f_R^{bb}-f_R^{aa}-g_R^2\tilde{\Delta}\ln\tilde{\Delta}^2]\;, \quad (26)$$

where $f_R^{\sigma\sigma'}$ and g_R are the renormalized couplings. Since up to this order we do not distinguish between bare and renormalized couplings, the combination of this result with the definition (3) immediately leads to Eq. (1).

IV. SUMMARY AND CONCLUSIONS

In summary, we have presented a fully self-consistent calculation of the true FS of two coupled metallic chains in the regime where the Luttinger liquid fixed point is stable. Our final result for the renormalized FS is given by the solution of the transcendental equation (1) shown in Fig. 2, which is perhaps the simplest example for an explicit solution of the self-consistency problem for the true FS discussed by Nozières[11]. We find that for small interchain hopping $t_\perp$ even weak interchain backscattering can lead to a strong reduction of the distance between the Fermi points associated with the bonding and the antibonding band. It is tempting to speculate that in higher dimensions a similar smoothing effect of a corrugated FS stabilizes a strongly correlated non-Fermi liquid state. Our method is general and can be used to calculate the true FS in higher dimensions within the framework of the RG; of course, in this case the resulting integral equations can only be solved numerically.

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